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Scientific Puzzle Solving: Current Techniques and Applications

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Abstract

An automated assembling of shredded/torn documents (2D) or broken pottery (3D) will support philologists, archaeologists and forensic experts. Especially if the amount of fragments is large (up to 1000), the compilation will not be feasible for a human puzzle solver due to cost and time. Approaches to solve the task of fragmented objects or puzzles can be divided into shape based matching techniques (apictorial) or techniques that analyze also the visual content of the fragments (pictorial), e.g. either image features or texture based analysis are done. Depending on the application, shape matching techniques are suitable for entities of the puzzle problem with small numbers of pieces (e.g. up to 20) and they cannot be applied to applications with hundreds of fragments due to the complexity. Also, artefacts like broken and lost pieces or overlapping parts of fragments increase the error rate of shape based matching techniques. This paper presents an overview about puzzle-solving techniques and their applications. It will introduce to the main problems in solving puzzles and will also focus on applications for the assembling of fragmented (ancient) documents..

Key words: *Artefact Reconstruction, Document Reconstruction*

1 Introduction

Finding solutions for puzzles need a well-defined definition of the problem, since a wide variation and synonyms for the term "Puzzle" exists. Games like Rubik's Cube, Puzz3D, Crosswords, the Tower of Hanoi or jigsaws may all be seen as a kind of Puzzle, whereas different strategies may be used for solving these. The most common form when talking about puzzles are 2D pictorial cardboard puzzles (also known as jigsaw puzzles).
In history the idea of a pictorial puzzle goes back to John Spilsbury[1] who made a jigsaw puzzle out of a wooden map by cutting the borders of countries using a jigsaw. Spilsbury lived in London, working as an engraver and mapmaker[1]. The idea of his wooden jigsaw puzzles which he built in the 1760's, was to create an educational tool, which can be used by children to learn geography[2]. It is generally agreed, that this was the first jigsaw puzzle in history[1]. These puzzles made out of sheets of hardwood were followed by cardboard puzzles in the late 1800s and became a die-cut process in the 20th century[1].
The definition of puzzle in Merriam-Websters online dictionary is "a question, problem, or contrivance designed for testing ingenuity"[3]. To find approaches or algorithms to solve a puzzle, the "problem" has to be well-defined. Since torn or shredded documents belong to pictorial or apictorial jigsaw (see Section 2) puzzles, we have to formulate a definition of this issue. For instance Freeman defines jigsaw puzzles as an "arrangement of a set of given pieces into a single, well-fitting structure, with no gaps left between

[1] R. Tybon. *Generating Solutions to the Jigsaw Puzzle Problem*. PhD thesis, Griffith University, Australia, 2004.

[2] E. D. Demaine and M. L. Demaine. *Jigsaw puzzles, edge matching, and polyomino packing: Connections and complexity*. Graphs and Combinatorics, 23(1):195–208, 2007.

[3] Merriam Webster Online Dictionary, *Puzzle*. http://www.merriam-webster.com/dictionary/puzzle[2] (accessed Jannuary 2009).

adjacent pieces"[4]. A similiar definition is presented in the online Encyclopedia Britannica, where jigsaw puzzles are "any set of varied, irregularly shaped pieces that, when properly assembled, form a picture or map"[5]. Related to documents we additionally have to define that gaps are possible in the case of broken and lost border pieces (for ancient documents, it is also possible that even entire fragments are lost).

Although puzzles are well known popular games, the automated solving is of great interest in different scientific areas like archaeology or forensics[6]. In this area artefacts of e.g. broken pottery[7] have to be reconstructed as well as shredded or torn documents in the e.g. forensics[6]. One of the first who dealt with the automatic solution of jigsaw puzzles in the scientific literature was Freeman and Garder[4].

This paper is organized as follows: Section 2 classifies the different types of puzzles. In Section 3 the state of the art of the solution of 2D puzzles is discussed and also a survey of current archaeological applications is presented, which is followed by a Conclusion 4.

2 Puzzle Classification and Applications

Generally puzzles can be divided into 2D and 3D puzzles. In this paper 2D puzzles are referenced to any kind of jigsaw puzzles. For instance crossword puzzles can also be considered as 2D puzzles, but are not discussed in this paper. Furthermore 2D jigsaw puzzles can be divided into pictorial and apictorial puzzles[4]. In apictorial puzzles only the shape of the puzzle pieces can be considered as information to assemble a single fitting structure. Compared to pictorial puzzles the shape as well as the information printed on the pieces (e.g. in terms of printed pictures or text) are accounted to find the correct solution. A variation of pictorial puzzles are edge matching puzzles[8], where all puzzle pieces have the same shape (e.g. squares). As a result, only the texture information of the pieces can solve the problem.

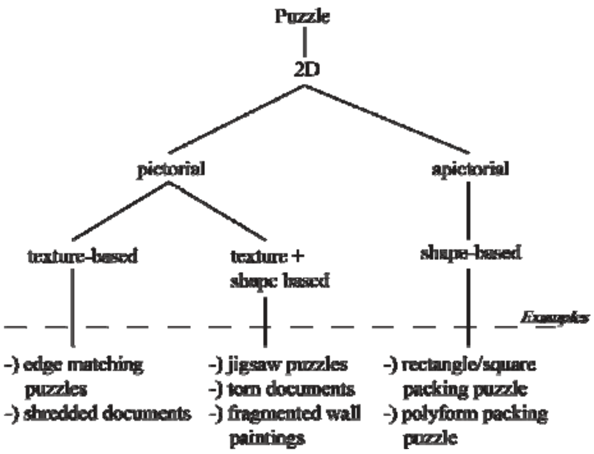

Figure 1: Types of 2D puzzles

In Demaine[8] all these types of puzzles are discussed, and it is shown, that these problems are NP-complete. Figure 1 shows the classification of the mentioned puzzles. Puzzles and the solving of them are important in different applications and sciences like forensics[6]. Automated solving strategies become important, if shredded documents have to be reconstructed[6]. This is considered as pictorial puzzle, since the shape of the shredded documents is the same for all pieces (e.g. stripes). If the documents are torn manually also/or only the shape can be used to find a solution. Manually torn sheets of paper may also be seen as a kind of a 3D puzzle since it is

[4] H. Freeman and L. Garder. *Apictorial jigsaw puzzles: The computer solution of a problem in pattern recognition*. Electronic Computers, IEEE Transactions on, EC-13(2):118–127, April 1964.

[5] Britannica Encyclopedia, *Jigsaw Puzzle (search term)*. http://www.britannica.com/ (accessed Jannuary 2009).

[6] A. Ukovich and G. Ramponi. *Features for the reconstruction of shredded notebook paper*. *Image Processing*, ICIP05. IEEE International Conference on, 3:III–93–6, 2005.

[7] M. Kampel and R. Sablatnig. *On 3d mosaicing of rotationally symmetric ceramic fragments*. In Proc. of 17th International Conference on Pattern Recognition, Cambridge, UK, volume 2, pages 265– 268. IEEE Computer Society, 2004.

[8] E. D. Demaine and M. L. Demaine. *Jigsaw puzzles, edge matching, and polyomino packing: Connections and complexity*. *Graphs and Combinatorics*, 23(1):195–208, 2007.

possible, that it tears in different layers of the paper. When paper tears in different layers the matching of different pieces of one sheet of paper can not be done by matching the outer border of the fragments. As a result a correct solution of this puzzle has overlapping parts.

A different problem arises, if fragments belong to different pages or even to different manuscripts. In this case it is possible to reduce the search space for the matching algorithm, if the fragments are clustered to the belonging manuscript or to the correct pages. To realize a clustering the visual (pictorial) content of the fragments must be analyzed (e.g. texture analysis to separate white paper from ruled paper).

Figure 2: Fragments of an ancient document

Within the project "*The Sinaitic Glagolitic Sacramentary (Euchologium) Fragments*" ancient documents have been digitized[9]. The most important manuscript in this project is the Missale Sinaiticum (Cod. Sin. slav. 5/N, 11th century), belonging to the classical Old Church Slavonic (OCS) canon. The folios of the manuscripts are degraded due to water influence (parts of the text are washed out, possible degradations of parchment, see Fuchs and Meinert[10]). Figure 2 shows broken fragments of the digitized manuscripts. It is not known a priori to which folio or even to which manuscript the individual fragments belong. Furthermore, it is not known, if parts are missing and also holes (interior boundaries) can appear.

A similiar puzzle problem is published in Papaodysseus et al.[11] In this paper fragmented wall paintings (1600 b.c.) are reconstructed by digitizing the surface of the fragments and matching the contours.

In Koller[12] the reconstruction of the Severan Marble Plan of Rome, or Forma Urbis Romae is treated. This map of Rome was destroyed in the 5th century and about more than 1000 fragments have been archived[12]. Additional to photographs of the top and bottom surface, a 3D model of all fragments has been scanned, and a database of all fragments was developed. A boundary incision matching method was developed, which uses topographic features in addition to the boundary information[12]. A matching algorithm simply based on contour matching did not lead to a solution. This project points out the archaeological importance of this topic.

A 2D map assembling (French cadastral map) approach is presented in Viglino and Guigues[13]. The proposed approach can also handle puzzles when fragments are missing. In[14] a profile based

[9] Kleber et al. *Multispectral Acquisition and Analysis of Ancient Documents*. In Proc. of the 14th International Conference on Virtual Systems and MultiMedia (VSMM 2008), Dedicated to Cultural Heritage - Project Papers, pages 184–191, Limassol, Cyprus, 2008.

[10] R. Fuchs, C. Meinert, and J. Schrempf. *Pergament: Geschichte, Material, Konservierung, Restaurierung*. Anton Siegl, Fachhochschule Köln, 2001.

[11] Papaodysseus et al. *Contour-shape based reconstruction of fragmented, 1600 bc wall paintings*. Signal Processing, IEEE Transactions on, 50(6):1277–1288, 2002.

[12] Koller. *Virtual Archaeology and Computer-Aided Reconstruction of the Severan Marple Plan*. In Beyond Illustration: 2D and 3D Digital Technologies as Tools for Discovery in Archaeology, British Archaeological Reports International Series, pages 125–134. Archaeopress, 2008.

[13] J.-M. Viglino and L. Guigues. *Cadastre map assembling: a puzzle game resolution*. Document Analysis and Recognition, 2001. Proceedings. Sixth International Conference on, pages 1235–1239, 2001

[14] M. Kampel and R. Sablatnig. *Profile based pottery reconstruction*. In Proc. of IEEE/CVPR Workshop on Applications of Computer Vision in Archaeology, pages CD–ROM, Madison, Wisconsin, USA, 2003.

3D mosaicing of archaeological pottery is presented. While 3D puzzles are not treated in this paper, the classification shown in Figure 1 can also be applied to 3D puzzles, whereas shape must be interpreted as 3D shape.

3 Current Applications to Reconstruct Fragmented Objects

According to the classification of puzzle types (see Figure 1) reconstruction methods are summarized in the next subsections (only 2D puzzles are considered).

3.1 Apictorial Reconstruction Methods

The classical approach to solve puzzles as well as to reconstruct torn paper is to use the information of the shape. Freeman and Garder[15] have been one of the first who discussed the problem of apictorial jigsaw puzzles. They have defined the following characteristics for jigsaw puzzles (for a detailed description see Freeman and Garder[15]):

- *Orientation*: usually not known a priori
- *Connectedness*: defines the possibility of "holes" and missing pieces
- *Exterior Boundary*: defines, if the shape of the outer boundary is known a priori
- *Uniqueness*: is characteristic for a puzzle, if only one or more solutions can exist
- *Radiality*: defines the kind of junctions of puzzle pieces

The shape of the puzzle pieces is presented as chain code, and features like e.g. the length of a side of a piece are used to reduce the search space. This method was tested with a 9 piece sample puzzle.

Burdea et al.[16] also use shape matching for the assembly of jigsaw puzzles. Furthermore, they use the characteristics of jigsaw puzzles that the frame (defined as *Exterior Boundary* by Freeman and Gardner) is a priori known (rectangle shape). For the automated assembly they use a "common human heuristic"[16], which means that the frame is assembled, and afterwards the interior of the puzzle. This is done since the frame pieces can be identified by one or two straight line sides. To test the algorithm two puzzles with 104 pieces were choosen.

As presented in the previous section Papaodysseus et al.[17] deal with the reconstruction of fragmented wall paintings. Compared to the classical jigsaw it is pointed out that facts like frame pieces, a priori knowledge of the shape or size of the pieces do not exist. Due to environmental influences (e.g. erosion) small missing fragments are possible, such that gaps between matching fragments are introduced (defined as Connectedness by Freeman and Gardner).

Additionally, it is stated that no uniqueness in this kind of archaeological puzzle exists. A local-curve-matching procedure that can handle gaps was developed and tested on two sets with 262 fragments and 936 fragments. Although the reconstruction of the set of 262 fragments worked, the authors stated that to solve a larger set additional parameters like color or thematic content continuation have to be considered.

[15] H. Freeman and L. Garder. *Apictorial jigsaw puzzles: The computer solution of a problem*. In pattern recognition. Electronic Computers, IEEE Transactions on, EC-13(2):118–127, 1964.

[16] B. Burdea and H. Wolfson. *Solving jigsaw puzzles by a robot*. Robotics and Automation, IEEE Transactions on, 5(6):752–764, 1989.

[17] C. Papaodysseus et al. *Contour-shape based reconstruction of fragmented, 1600 bc wall paintings*. Signal Processing, IEEE Transactions on, 50(6):1277–1288, 2002.

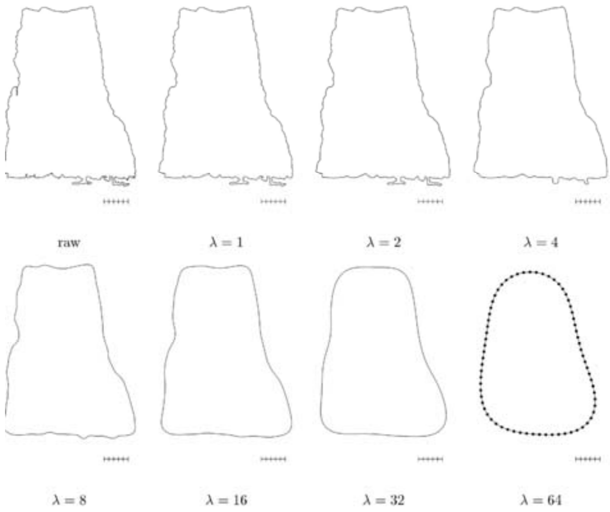

Figure 3: Multiscale representation of a single fragment, courtesy of Leitao and Stolfi[18]

Another approach dealing with the reconstruction of archaeological artefacts is presented in da Gama Leitao and Stolfi[18]. A multiscale-matching method is developed and tested on a 112 piece fragmented unglazed ceramic tile. To find initial matching candidates a fast search on a coarse scale is done. Afterwards a matching on a finer scale is done by "comparing only the most promising candidates found at the previous scale". In Figure 3 the representation of a fragment at different scales is shown. The computational cost of the matching is $O(N^2 L \log L)$. However, the authors state that the multiscale-matching approach is suited for granulated material and has to be tested on different materials like glass, which provide sharper edges. In da Gama Leitao and Stolfi[19] the authors raise the question, if it is possible to match fragments by their shape due the information content of fracture lines for a large number of fragments (large here means up to 1,000s of fragments[19]). Although the result is dependend on the material (the authors again use ceramic tiles) and noise (e.g. erosion), it is stated that "there is enough information in typical fragment outlines to solve even very large instances of the reassembly problem", which is due the fact that "the amount of information required grows very slowly (logarithmically) with the size of the problem"[19].

3.2 Pictorial Reconstruction Methods

To enhance the search of matching pieces additional features can be taken into account. In Chung et al.[20] the shape as well as sample colors along the border of the pieces are calculated. It was tested on 6 different jigsaw puzzles with 12 up to 54 pieces. These test sets have all been correctly assembled and it is shown, that uses of chromatic information leads to a "significantly aid in solving the partial boundary matching problem"[20].

A texture based method where the shape of the pieces is not considered is presented in Sagiroglu and Ercil[21]. The algorithm was tested on fragments of the Forma Urbis Romae dataset consisting of 13 pieces (broken artificially) and on a 21 pieces set of fragmented ceramic tiles. In the second dataset the ceramic tiles come from two different tile sets. To assemble 2 fragments the border of the fragments is extended using inpainting and texture synthesis techniques. Image features of the pieces are calculated (mean and variance) that are used for the matching of 2 pieces by using FFT. For a detailed description see Sagiroglu and Ercil[21].

Nielsen et al.[22] solve classical jigsaw puzzles by using image features (again without using the shape of a puzzle piece). One pixel wide edge strips of one side of the puzzle pieces (are assumed

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[18] H. C. da Gama Leitao and J. Stolfi. *A multiscale method for the reassembly of two-dimensional fragmented objects*. IEEE Trans. Pattern Anal. Mach. Intell., 24(9):1239–1251, 2002.

[19] H. C. da Gama Leitao and J. Stolfi. *Measuring the information content of fracture lines.* Int. J. Comput. Vision, 65(3):163–174, 2005.

[20] M. G. Chung, M. Fleck, and D. Forsyth. *Jigsaw puzzle solver using shape and color.* Signal Processing Proceedings, 1998. ICSP '98. 1998 Fourth International Conference on, 2:877–880, 1998.

[21] M. Sagiroglu and A. Ercil. *A texture based matching approach for automated assembly of puzzles*. Pattern Recognition, 2006. ICPR 2006. 18th International Conference on, 3:1036–1041, 2006.

[22] T. R. Nielsen, P. Drewsen, and K. Hansen. *Solving jigsaw puzzles using image features.* Pattern Recogn. Lett., 29(14):1924–1933, 2008.

to have a rectangular shape) are extracted, and an edge detector is used to calculate the similarity of the two stripes that are aligned side by side. To assemble the entire puzzle an adaption of the proposed algorithm in Burdea and Wolfson[23] is implemented. The algorithm was tested on two synthetically generated puzzles with up to 504 pieces. The puzzles consisting of 100 pieces have been correctly assembled, whereas in the 320-piece-puzzles up to 7,2% pieces are incorrectly placed.

In Yao and Shao[24] a shape matching approach is combined with an image merging process. First all puzzle pieces (belonging to a canonical jigsaw puzzle[25]) are classified as a defined type and the four sides of every piece are extracted by using dominant points. Using the shape matching part side-by-side candidate pieces are determined. The "true matching edge"[24] is decided by calculating the integration degree (separability of image features) of an image. As image features the R-, G- and B-components are used. Puzzles with a dimension up to 16 pieces are tested.

4 Conclusion

In this paper different types of puzzles are defined, and a definition of a puzzle related to torn documents was presented. Additionally, published applications of puzzles and their solution in cultural heritage are summarized. Finally, methods to solve different types of this problem are introduced.

Although shape matching techniques work for documents with a small number of fragments (e.g. up to 20) and da Gama Leitao and Stolfi[26] states that this technique is theoretically also suitable for large instances (up to 1,000s of fragments), aspects like erosion of the border and missing parts make it essential to expand the matching by using additionally content based features. As an example the work of Sagiroglu and Ercil[27] (see Section 3.2) was only tested on fragments with sharp edges (no information is lost). This approach will fail in the case of torn paper. Therefore, for large instances information of the texture as well as the shape has to be considered.

As a future work the authors will work out different matching techniques working best for torn documents with a large number of fragments. Additionally, methods to handle also constraints like holes in fragments or missing fragments will be treated as well as overlapping parts. In the case of textual documents additional features like the skew will reduce the matching possibilities and enhance the assembling, too.

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[24] F.-H. Yao and G.-F. Shao. *A shape and image merging technique to solve jigsaw puzzles*. Pattern Recogn. Lett., 24(12):1819–1835, 2003.

[25] R. Tybon. *Generating Solutions to the Jigsaw Puzzle Problem*. PhD thesis, Griffith University, Australia, 2004.

[26] H. C. da Gama Leitao and J. Stolfi. *Measuring the information content of fracture lines*. Int. J. Comput. Vision, 65(3):163–174, 2005

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